Bridging Physics and Learning: A Hybrid Deep Model for Structural Deformation Prediction

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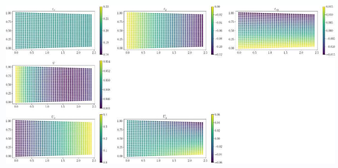
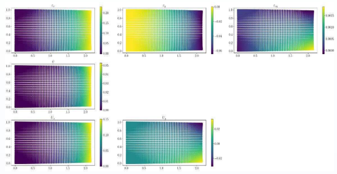
INTRODUCTION

Simulating structural deformation is fundamental to many engineering applications. While traditional methods such as the **Finite Element Method (FEM)** are well-established, they can be computationally intensive and inflexible in real-time or data-scarce scenarios.

This work presents a mesh-free, physics-informed neural approach to 2D elasticity, using a **Deep Energy Method (DEM)** that embeds physical principles—such as strain energy, force balance, and boundary constraints—directly into the loss function of a neural network. The goal is to learn the displacement field $u(x, y) = (u_x, u_y)$ across the domain by minimizing total potential energy, **without requiring labeled data**.

We explore various loss combinations, sampling strategies, numerical integration schemes, and compare classical dense networks with a more efficient **Kolmogorov–Arnold Network (KAN)** implementation.

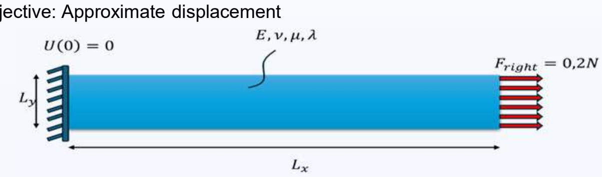




APPROACHES

Problem Definition

- A 2D elastic beam under uniaxial traction
- Boundary conditions:
  - left edge fixed (Dirichlet)
  - right edge loaded (Neumann)
- Assumptions: Plane stress, Hookean or Neo-Hookean material law
- Objective: Approximate displacement



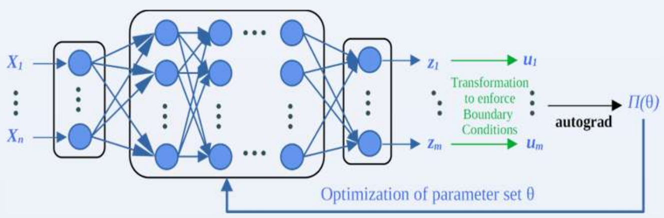
Learning Objective

The neural network $u_\theta = (u_x, u_y)$ is trained to minimize a physics-based energy functional:

$$\mathcal{L}_{total} = \underbrace{\mathcal{L}_{strain}}_{\text{internal energy}} + \underbrace{\mathcal{L}_{traction}}_{\text{external load}} + \underbrace{\mathcal{L}_{div}}_{\text{force balance}} + \underbrace{\mathcal{L}_{bc}}_{\text{Dirichlet constraints}}$$

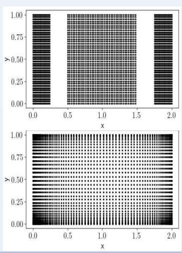
Model Architectures

Network	Description
FCNN	Fully-connected with Tanh or custom activations
KAN	Sparse functional representation with trainable basis functions (custom-built, efficient)



Numerical Enhancements

- Integration: Simpson's rule over regular mesh
- Sampling:
  - Uniform grid
  - Adaptive near-boundary grid (Type 2, Type 3)
- Force scheduling: Gradual load increase to enhance training stability



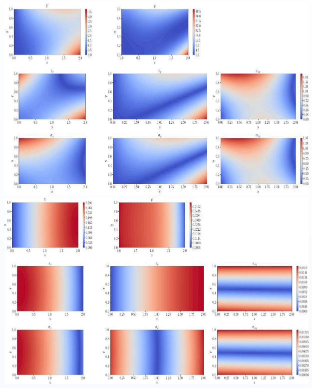
RESULTS

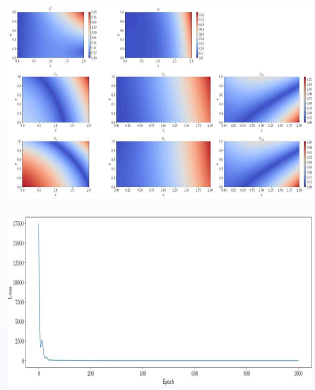
Quantitative Summary

Model	Sampling	Loss Type	Rel. L^2- Error	Training Stability
FCNN	Uniform	Type 011	~6.5%	Oscillatory
FCNN	Type 2	Type 111	~3.2%	Stable
KAN	Type 2	Type 011	~2.7%	Fast convergence

Highlights

- **KAN** achieves superior accuracy with fewer parameters and faster training
- Models using **strain + divergence + boundary losses** outperform simpler configurations
- **Sampling near boundaries** improves accuracy without increasing training time
- Simpson's integration provides smoother convergence than trapezoidal rule





CONCLUSIONS

- Deep energy-based neural networks can effectively approximate solutions to elasticity problems without labeled supervision
- Embedding physical laws into the loss function yields robust generalization and physically consistent predictions
- The custom KAN architecture is a compact and efficient alternative to conventional dense networks
- Sampling strategy and integration scheme significantly impact accuracy and convergence

Outlook

- ☐ **Material complexity**: Extend to Neo-Hookean models with large deformations
- ☐ **Higher-dimensional cases**: 3D geometries and anisotropic materials
- ☐ **Stress-based loss**: Explore mixed deep energy methods (mDEM) to predict stress tensors directly
- ☐ **Engineering deployment**: Use in digital twins and lightweight embedded simulators
- ☐ **Benchmarking**: Compare performance with commercial FEM tools (e.g. Abaqus)

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