A Variational Network for MR Image Reconstruction

Jiaxin Zhang

Friedrich-Alexander-Universität Erlangen Nürnberg

Email: jiaxin.zhang@fau.de

AI 2025 SUMMER SCHOOL

ai.uni-jena.de

INTRODUCTION

Magnetic Resonance Imaging (MRI) is a wildly used tomography technique with high sensitivity in soft tissue and brain tissue. Comparing to other radioactive examinations, MRI doesn't have harmful radiation. Comparing to the almost instance x-ray or fast CT, it needs **longer time** to acquire image data and later a rather complex algorithm to reconstruct images.

MR signal is collected by exciting the spins in the body and collect data from the receive coil. Due to conventional application with phase encoding technique, the acquired data, which is **Fourier-like format**, is in kspace and need further process, usually through a Discrete Fourier Transform (DFT) to reconstruct the image. The image resolution depends on the voxel size. Keep the field of view static, the more voxels one slice have, the higher resolution the image obtain, but more time needs to be spent. To **reduce the examination time** while preserve the image quality, a method called **compressed sensing (CS)** is used in conventional examination.

CS is an iterative algorithm, with irregularly sampled MRI data, the reconstruct image will have noise like artifact. By removing those artifact, it can accelerate the acquisition by a factor of 10. Lately, with deep learning, the image acquisition time can reduce to 50%[1].

This poster introduces a method called **"Variational Network (VN)"** which combines CS with deep learning method to enhance image quality to get a higher Structural Similarity Index (SSIM) and higher peak signal to noise ratio (PSNR)[2]. An experiment was performed and achieved a SSIM of 88.01% and PSNR of 32.45 dB. All data is obtained from NYU fastMRI Initiative database (fastmri.med.nyu.edu)[4][5].

METHODS

In MRI reconstruction we oftentimes deal with **complex number**. First we map the complex image data $\tilde{u}$ to **real valued number** as following:

$$\tilde{\boldsymbol{u}} = \boldsymbol{u}_{re} + j\boldsymbol{u}_{im} \in \mathbb{C}^N \Leftrightarrow \boldsymbol{u} = (\boldsymbol{u}_{re}, \boldsymbol{u}_{im}) \in \mathbb{R}^{2N}$$

We consider the ill-posed inverse problem of finding a reconstructed image $\boldsymbol{u} \in \mathbb{R}^{2N}$ satisfies the equation:

$$\boldsymbol{A}\boldsymbol{u} = \hat{\boldsymbol{f}}$$

Where $\hat{\boldsymbol{f}}$ is fully sampled kspace and $\boldsymbol{f}$ is undersampled kspace. A is an linear operator with pixel-wise multiplication with coil sensitivity maps, Fourier transforms and undersampling. We minimize the **least squares error** to compute $\boldsymbol{u}$.

$$\min_{\boldsymbol{u}} \frac{1}{2}\|\boldsymbol{A}\boldsymbol{u} - \hat{\boldsymbol{f}}\|_2$$

In practice we lack information of $\hat{\boldsymbol{f}}$, thus we perform a **gradient descent (GD)** on the least squares error equation. This led to an iterative method called Landweber method, which can be written as

$$\boldsymbol{u}^{t+1} = \boldsymbol{u}^t - \alpha^t \boldsymbol{A}^*(\boldsymbol{A}\boldsymbol{u}^t - \boldsymbol{f}), \qquad t \geq 0$$

Where α^t is step sizes and $\boldsymbol{A}^*$ is the adjoint linear sampling operator. To prevent over-fitting, we extend the least square problem by **an additional regularization term** $\mathcal{R}(u)$.

$$\min_{\boldsymbol{u}} \left\{ \mathcal{R}(\boldsymbol{u}) + \frac{\lambda}{2}\|\boldsymbol{A}\boldsymbol{u} - \boldsymbol{f}\|_2^2 \right\}$$

Fields of Experts model is used in the regularization term, with a trainable reaction diffusion approach that **performs early stopping** on gradient, the iteration is written as:

$$\boldsymbol{u}^{t+1} = \boldsymbol{u}^t - \sum_{i=1}^{N_k} \left(\boldsymbol{K}_i^t\right)^T \boldsymbol{\Phi}_i^{t\prime}\left(\boldsymbol{K}_i^t \boldsymbol{u}^t\right) + \lambda^t \boldsymbol{A}^*(\boldsymbol{A}\boldsymbol{u}^t - \boldsymbol{f}), \qquad 0 \leq t \leq T-1$$

Where $\boldsymbol{K}_i^t$ models convolutions with filter kernel and $\boldsymbol{\Phi} = (\phi_1, \phi_2, \dots, \phi_N)$ is the non-linear potential functions. Each iteration can be seen as one step in the network.

Now we can convert this equation into network as shown in Fig 1 with the following parameters: filter kernels k_i^t, activation functions $\Phi_i^{t\prime}$, and data term weights λ^t. The filter should be zero-mean and restricted to unit-sphere to avoid scaling problem of the activation functions. Activation functions $\Phi_i^{t\prime}$ is set to a weighted Gaussian radial basis functions (RBFs), which can be expressed as:

$$\phi_i^{t\prime}(z) = \sum_{j=1}^{N_w} w_{ij}^t \exp\left(-\frac{(z-\mu_j)^2}{2\sigma^2}\right)$$

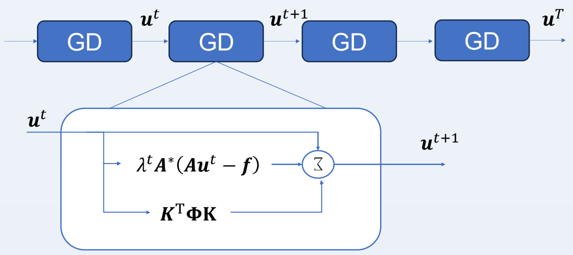

Fig 1: Architecture of VN

The training process is aimed to find **optimal parameter set**. To achieve that we need to minimize the loss on the dataset. A common approach is to **use mean-squared error** (MSE). Since normally we use the magnitude images, MSE (ϵ-smoothed) of absolute value was defined as:

$$\mathcal{L}(\theta) = \min_{\theta} \frac{1}{2S}\sum_{s=1}^{S} \left\| \left|\boldsymbol{u}_s^T(\theta)\right|_\epsilon - |\boldsymbol{g}_s|_\epsilon \right\|_2^2, \qquad |\boldsymbol{x}|_\epsilon = \sqrt{\boldsymbol{x}_{re}^2 + \boldsymbol{x}_{im}^2 + \epsilon}$$

Where $\boldsymbol{g}$ is the artifact-free image, $\boldsymbol{u}$ is reconstructed image, S is a set of images and $\theta = \{w_{ij}^t, \boldsymbol{k}_i^t, \lambda^t\}$ is the parameter set.

To solve this, an optimizer called Inertial Incremental Proximal Gradient (IIPG) is used. It's rooted in Inertial Proximal Alternating Linearized Minimization (IPALM) algorithm[3]. The optimizer require the **gradient w.r.t parameters** and the **value of the loss function**. The gradient can be computed by back propagation using chain rule:

$$\frac{\partial \mathcal{L}(\theta)}{\partial \theta^t} = \frac{\partial \mathcal{L}(\theta)}{\partial \boldsymbol{u}^T} \cdot \frac{\partial \boldsymbol{u}^T}{\partial \boldsymbol{u}^{T-1}} \cdots \frac{\partial \boldsymbol{u}^{t+2}}{\partial \boldsymbol{u}^{t+1}} \cdot \frac{\partial \boldsymbol{u}^{t+1}}{\partial \theta^t}$$

By utilizing those optimized parameters, we can reconstruct the images using undersampled kspace data.

EXPERIMENT

The experiment used fastMRI Dataset[4][5] multicoil data with 15 coils. The undersampled kspace is shown in Fig 2. The center of the kspace is fully preserved, because that's the part contains most of the contrast information. High frequency part is undersampled to achieve less scanning time. From there 150 objects divided to 5 batches, are used for training, 20 for validation and 19 for testing. Total trainable parameters are 9.6 million. With 25 epochs in training, the neural network achieve a overall SSIM of 88.01% and PSNR of 32.45 dB. One of the reconstructed object is shown in Fig 3.

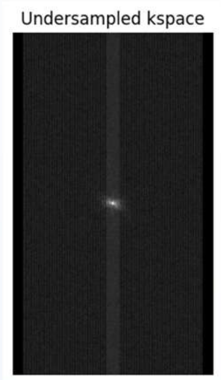

Fig 2: undersampled kspace

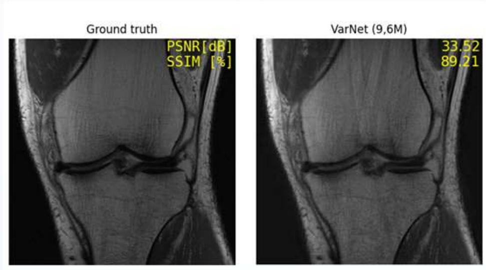

Fig 3: test result

We can see in the reconstructed image there are still some aliasing alike those we can observe in parallel imaging in the middle. And also some noise like artifact is shown in the image.

Moreover, VN is a rather **high time cost** method because of the iteration and large trainable parameters. Recently some studies implement some distillation network to reduce the time. A group in China achieved using only **1/10 of the time** with almost the same SSIM and PSNR[6].

REFERENCES

[1]. Dratsch, Thomas, et al. "Reconstruction of 3D knee MRI using deep learning and compressed sensing: a validation study on healthy volunteers." *European radiology experimental* 8.1 (2024): 47.

[2]. Hammernik, Kerstin, et al. "Learning a variational network for reconstruction of accelerated MRI data." Magnetic resonance in medicine 79.6 (2018): 3055-3071.

[3]. Pock, Thomas, and Shoham Sabach. "Inertial proximal alternating linearized minimization (iPALM) for nonconvex and nonsmooth problems." *SIAM journal on imaging sciences* 9.4 (2016): 1756-1787.

[4]. Zbontar, J., et al. "fastMRI: An open dataset and benchmarks for accelerated MRI. arXiv 2018." *arXiv preprint arXiv:1811.08839* (1811).

[5]. Knoll, Florian, et al. "fastMRI: A publicly available raw k-space and DICOM dataset of knee images for accelerated MR image reconstruction using machine learning." *Radiology: Artificial Intelligence* 2.1 (2020): e190007.

[6]. Wu, Zhengliang, and Xuesong Li. "Adaptive knowledge distillation for high-quality unsupervised mri reconstruction with model-driven priors." *IEEE Journal of Biomedical and Health Informatics* 28.6 (2024): 3571-3582.

ACKNOWLEDGEMENTS and CONTACT

Code is rooted in an exercise of the course "Computational MRI" held by Prof. Knoll, with the co-author Jinho Kim. Email: jinho.kim@fau.de.

Supervisor: Martin Bücker, Email: martin.buecker@uni-jena.de.